

Paper Reference 9FM0/3A
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced

PAPER 3A: Further Pure Mathematics 1

Thursday 22 June 2023 – Afternoon

Time: 1 hour 30 minutes

YOU MUST HAVE

**Mathematical Formulae and Statistical Tables (Green),
calculator**

YOU WILL BE GIVEN

**Diagram Booklet
Answer Booklet**

X72796RA

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear.

Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

**There are 7 questions in this Question Paper.
The total mark for this paper is 75.**

**The marks for EACH question are shown in brackets
– use this as a guide as to how much time to spend on
each question.**

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. (a) Use Simpson's rule with 4 intervals to find an estimate for

$$\int_0^2 e^{\sin^2 x} dx$$

Give your answer to 3 significant figures.

(4 marks)

Given that

$$\int_0^2 e^{\sin^2 x} dx = 3.855 \text{ to 4 significant figures,}$$

- (b) comment on the accuracy of your answer to part (a).

(1 mark)

(Total for Question 1 is 5 marks)

2. The vertical height, h metres, above horizontal ground, of a passenger on a fairground ride, t seconds after the ride starts, where $t \leq 5$, is modelled by the differential equation

$$t^2 \frac{d^2h}{dt^2} - 2t \frac{dh}{dt} + 2h = t^3 \quad (\text{I})$$

- (a) Given that $t = e^x$, show that

$$(i) \quad t \frac{dh}{dt} = \frac{dh}{dx}$$

$$(ii) \quad t^2 \frac{d^2h}{dt^2} = \frac{d^2h}{dx^2} - \frac{dh}{dx}$$

(4 marks)

(continued on the next page)

2. continued.

Remember: $t^2 \frac{d^2h}{dt^2} - 2t \frac{dh}{dt} + 2h = t^3$ (I)

(b) Hence show that the transformation $t = e^x$ transforms equation (I) into the equation

$$\frac{d^2h}{dx^2} - 3 \frac{dh}{dx} + 2h = e^{3x}$$

(1 mark)

(c) Hence show that

$$h = At + Bt^2 + \frac{1}{2}t^3$$

where **A** and **B** are constants.

(6 marks)

(continued on the next page)

2. continued.

Given that when

$t = 1$, $h = 2.5$ and when

$$t = 2, \frac{dh}{dt} = -1$$

- (d) determine the height of the passenger above the ground **5** seconds after the start of the ride.
(5 marks)

(Total for Question 2 is 16 marks)

3. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Refer to the diagram for Question 3 in the Diagram Booklet.

It shows a sketch of the curve with equation

$$y = \frac{x^2 - 2x - 24}{|x + 6|}$$

and the line with equation

$$y = 5 - 4x$$

Use algebra to determine the values of x for which

$$\frac{x^2 - 2x - 24}{|x + 6|} < 5 - 4x$$

(Total for Question 3 is 7 marks)

4. The ellipse E has equation

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

(a) Determine the exact value of the eccentricity of E

(2 marks)

The points $P(4 \cos \theta, 3 \sin \theta)$ and $Q(4 \cos \theta, -3 \sin \theta)$ lie on E where $0 < \theta < \frac{\pi}{2}$

The line L_1 is the normal to E at the point P

(b) Use calculus to show that L_1 has equation

$$4x \sin \theta - 3y \cos \theta = 7 \sin \theta \cos \theta$$

(4 marks)

(continued on the next page)

4. continued.

The line L_2 passes through the origin and the point Q

The lines L_1 and L_2 intersect at the point R

(c) Determine, in simplest form, the coordinates of R

(4 marks)

(d) Hence show that, as θ varies, R lies on an ellipse which has the same eccentricity as ellipse E

(2 marks)

(Total for Question 4 is 12 marks)

5. (a) Show that the substitution

$t = \tan\left(\frac{x}{2}\right)$ transforms the integral

$$\int \frac{1}{2 \sin x - \cos x + 5} dx$$

into the integral

$$\int \frac{1}{3t^2 + 2t + 2} dt$$

(4 marks)

- (b) Hence determine

$$\int \frac{1}{2 \sin x - \cos x + 5} dx$$

(4 marks)

(Total for Question 5 is 8 marks)

6. $y = \ln (e^{2x} \cos 3x) \quad -\frac{1}{2} < x < \frac{1}{2}$

(a) Show that

$$\frac{dy}{dx} = 2 - 3 \tan 3x$$

(2 marks)

(b) Determine

$$\frac{d^4 y}{dx^4}$$

(3 marks)

(c) Hence determine the first 3 non-zero terms in ascending powers of x of the Maclaurin series expansion of $\ln (e^{2x} \cos 3x)$, giving each coefficient in simplest form.

(3 marks)

(continued on the next page)

6. continued.

- (d) Use the Maclaurin series expansion for $\ln(1 + x)$ to write down the first 4 non-zero terms in ascending powers of x of the Maclaurin series expansion of $\ln(1 + kx)$, where k is a constant.

(1 mark)

- (e) Hence determine the value of k for which

$$\lim_{x \rightarrow 0} \left(\frac{1}{x^2} \ln \frac{e^{2x} \cos 3x}{1 + kx} \right)$$

exists.

(3 marks)

(Total for Question 6 is 12 marks)

7. With respect to a fixed origin **O** the point **A** has coordinates **(3, 6, 5)** and the line **L** has equation

$$\left(\underline{r} - (12\underline{i} + 30\underline{j} + 39\underline{k}) \right) \times (7\underline{i} + 13\underline{j} + 24\underline{k}) = \underline{0}$$

The points **B** and **C** lie on **L** such that

$$AB = AC = 15$$

Given that **A** does not lie on **L** and that the **x** coordinate of **B** is negative,

- (a) determine the coordinates of **B** and the coordinates of **C**

(4 marks)

- (b) Hence determine a Cartesian equation of the plane containing the points **A**, **B** and **C**

(3 marks)

(continued on the next page)

7. continued.

The point **D** has coordinates $(-2, 1, \alpha)$, where α is a constant.

Given that the volume of the tetrahedron **ABCD** is 147

(c) determine the possible values of α
(4 marks)

Given that $\alpha > 0$

(d) determine the shortest distance between the line **L** and the line passing through the points **A** and **D**, giving your answer to 2 significant figures.
(4 marks)

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
